ABSTRACT

of the dissertation submitted for the degree of Doctor of Philosophy (PhD)
in the specialty «6D060400 – Physics»

«Investigation of some integrable generalized nonlinear Schrödinger-type equations and their spin systems»

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Relevance of the topic

Soliton solutions of nonlinear Schrödinger-type equations and their generalizations play an important role in describing stable localized wave structures formed in various physical systems. Such structures arise as a result of the balance between dispersion and nonlinearity and retain their shape during propagation and interaction. These properties make solitons one of the key objects of study in nonlinear optics, plasma physics, magnetic media, hydrodynamics, and quantum condensed systems.

The need to study ultrashort pulses, high-energy wave packets, and multimode media requires more accurate mathematical models that go beyond the classical nonlinear Schrödinger equation (NLSE). One of such models is the Fokas-Lenells (FL) equation, which is an integrable NLSE and describes the propagation of ultrashort optical pulses in strongly nonlinear and dispersive media. Compared to the classical equation, it contains additional nonlinear and derivative terms describing self-steepening, nonlinear dispersion, and phase modulation.

Real physical systems often have complex spatial structures and vector nature of wave fields. In this regard, multidimensional and multicomponent generalizations of Schrödinger-type equations are of particular importance. Such models allow taking into account the interaction of several components and energy redistribution.

An important direction is also the study of higher-order generalized equations that take into account derivatives, higher-order dispersion, self-steepening of wave packets, and inhomogeneity of the medium. These effects play a key role in describing femtosecond pulses and extreme wave phenomena (breathers, «rogue waves»). Exact analytical solutions make it possible to better understand the physical nature of nonlinear regimes.

The connection of Schrödinger-type equations with spin systems via gauge transformations makes it possible to geometrically interpret solitons and study their stability. The study of dispersionless limits describing shock waves and sharp fronts is also important.

Thus, the relevance of the work is determined by the need for a comprehensive study of generalized multidimensional and multicomponent integrable models. The

obtained results contribute to a deeper understanding of the formation and evolution of localized waves and their application in optics, plasma, and quantum systems.

The aim of the dissertation

The theoretical investigation of generalized nonlinear Schrödinger-type equations and their spin systems, the determination of their solutions, and the study of their physical characteristics. To achieve this goal, the following specific objectives were set:

1. Finding solutions of generalized nonlinear Schrödinger-type equations;
2. Finding solutions of the spin system of the FL equation;
3. Constructing (2+1)-dimensional and two-component (2+1)-dimensional FL equations and their spin systems, and obtaining their solutions.

Object of research: integrable generalized nonlinear Schrödinger-type equations and their spin systems.

Subject of research: soliton solutions of nonlinear Schrödinger-type equations and the spin system of the FL equation.

Research methods: Darboux transformation, Hirota method, bilinearization, as well as gauge transformations to obtain an equivalent spin system.

Scientific novelty

1. Various solutions of generalized nonlinear Schrödinger-type equations and the spin system of the FL equation were obtained.
2. One- and two-component (2+1)-dimensional FL equations and their spin systems were constructed, and their one- and two-soliton solutions were obtained using the Darboux transformation.
3. Soliton surfaces induced by the FL equation, its dispersionless limits, and soliton solutions of the corresponding spin systems were proposed.

Theoretical and practical significance

The work is aimed at studying the integrable nature, soliton solutions, and geometric properties of generalized nonlinear Schrödinger-type equations and their spin systems. The results make it possible to combine methods of mathematical physics and differential geometry and can be applied in nonlinear optics, plasma physics, magnetic systems, and telecommunications.

Main provisions submitted for defense

1. Exact localized wave solutions for integrable nonlinear Schrödinger-type equations are obtained by means of the Darboux transformation method. Time-dependent dispersion $a_1(t)$, $a_3(t)$ and nonlinearity $a_2(t)$ coefficients control the localization, amplitude, stability, and trajectory of waves, while the spectral parameter ensures transitions between different types of localized waves.

2. The FL equation and its spin system are equivalent via a gauge transformation $\Phi = g^{-1}\Psi$ (where $g(x,t)$ a unitary matrix of size 2×2 is considered and $g = \Psi\big|_{z=\lambda_0}$),

This makes it possible to describe nonlinear wave processes by means of spin systems and preserves their integrable structure.

3. Based on the compatibility conditions, the (2+1)-dimensional and two-component (2+1)-dimensional FL equations are derived. The Lax pairs prove their integrability. The obtained soliton solutions describe the localization, stability, and elastic interaction of nonlinear waves.

Reliability and validity of the results

The reliability and validity of the research results are confirmed by publications in international peer-reviewed journals indexed in the Web of Science/Scopus databases, as well as in proceedings of international conferences, and by citations of these works by foreign scientists.

Author's contribution

The author directly participated in all stages of the dissertation research. The author carried out the formulation of the research problem, development of methods, performance of calculations, analysis of results, formulation of conclusions, and preparation of scientific texts. The author's contribution is clearly reflected in all published works and fully corresponds to the content of the dissertation.

Approbation of the work and publications

The main results of the work were presented and discussed at international scientific conferences: Lomonosov – 2018: the 14th International Scientific Conference of Students, Master's Students and Young Scientists (Astana, 2018); International Conference on Technology, Engineering and Science (Antalya, 2018, October 26–29); Science and Education – 2018: the 13th International Scientific Conference of Students and Young Scientists (Astana, 2018); International Conference "Current Problems of Analysis, Differential Equations and Algebra", October 16–19, 2019 (Astana, 2019); Science and Education – 2019: the 14th International Scientific Conference of Students and Young Scientists (Astana, 2019); 8th International Conference on Mathematical Modeling in Physical Sciences (Bratislava, 2019, August 26–29); 4th International Conference "Astrophysics, Gravity and Cosmology" (Astana, 2019, October 29–30); The 26th International Conference on Integrable Systems and Quantum Symmetries (Prague, 2019, July 8–12).

In addition, the main results of the work were reported and discussed at scientific seminars of the Department of General and Theoretical Physics of L.N. Gumilyov Eurasian National University.

Publications on the research topic

A total of 18 scientific works have been published based on the results of the dissertation research. Among them, 4 papers were published in high-impact international journals indexed in the Web of Science and Scopus databases. In addition, 2 papers were published in conference proceedings of journals indexed in

these databases, 1 paper in international conference proceedings, 5 papers in journals recommended by the Ministry of Science and Higher Education of the Republic of Kazakhstan, and 1 paper in other publications.

Main works:

1. **Zhassybayeva M.,** Shaikhova G., Yesmakhanova K., Myrzakulova Z. Darboux transformation and exact soliton solutions of the two-component (2+1)-dimensional Fokas-Lenells equation // Results in Physics. – 2025. – Vol. 79. – P 108517.

2. **Zhassybayeva M.,** Myrzakulova Z., Yesmakhanova K. Modified Fokas-Lenells equation: self-consistent sources and soliton solutions of the spin and (2+1)-dimensional models // Symmetry. – 2025. – Vol. 17, Issue 7. – P. 1961.

3. Nugmanova G., Bekova G., **Zhassybayeva M.,** Taishiyeva A., Yesmakhanova K., Myrzakulova Z. Ferromagnet-type system: Integrable flows of curves/surfaces, soliton solutions, and equivalence // Symmetry. – 2025. – Vol. 17, Issue 7. – P. 1041.

4. Taibi Z., Chaachoua Sameut H., **Zhassybayeva M.,** Sakthivinayagam P., Serikbayev N. Exploration of time-dependent dispersion and nonlinearity management in stabilization and transition of localized structures in nonlinear optical media // Symmetry. – 2025. – Vol. 17, Issue 12. – P. 2165.

5. **Zhassybayeva M.,** Yesmakhanova K. Soliton surfaces induced by the Fokas-Lenells equation // Journal of Physics: Conference Series. – 2019. – Vol. 1416, Issue 1. – P. 012042.

6. **Zhassybayeva M.,** Yesmakhanova K. Dispersionless limit of the (1+1)-dimensional Fokas-Lenells equation // Journal of Physics: Conference Series. – 2019. – Vol. 1391, Issue 1. – P. 012103.

7. **Zhassybayeva M.,** Yesmakhanova K. The Construction of the (2+1)-Dimensional Integrable Fokas-Lenells Equation and its Bilinear form by Hirota Method // International Conference on Technology, Engineering and Science. – 2018. – Vol. 4. – P. 61-66.

8. **Zhassybayeva M.,** Yesmakhanova K. Soliton solutions for the (2+1)-dimensional integrable Fokas-Lenells equation. News of the National Academy of Sciences of the Republic of Kazakhstan. – 2019. – Vol. 6, Issue 328. – P. 130-137.

9. **Zhassybayeva M.B.,** Nugmanova G.N. Spin system equivalent to the integrable Fokas-Lenells equation. Bulletin of L.N. Gumilyov Eurasian National University. – Astana, 2018. – No. 1(122). – P. 53–57.

10. **Zhassybayeva M.B.,** Yesmakhanova K.R. Darboux transformation and exact one-soliton solution of the Fokas-Lenells equation. Bulletin of L.N. Gumilyov Eurasian National University. – Nur-Sultan, 2020. – No. 3(132). – P. 8–13.

11. Myrzakulov E.M., **Zhassybayeva M.B.,** Kairbayeva M.A. Two-layer spin system associated with the two-component Fokas-Lenells equation. Bulletin of Sh. Ualikhanov Kokshetau University. – Kokshetau, 2022. – No. 1. – P. 4–14.

12. **Zhassybayeva M.B.,** Myrzakulova Zh.R. Darboux transformation and soliton solution of the derivative nonlinear Schrödinger equation of the first type. Bulletin of Toraighyrov University. – Pavlodar, 2025. – No. 3.

13. **Zhassybayeva M.B.** One-soliton solution of the M-LXXII equation. Proceedings of the XIII International Scientific Conference of Students and Young Scientists "Science and Education – 2018". – Astana, 2018. – P. 396–399.

14. **Zhassybayeva M.B.** Dispersionless limit of the (2+1)-dimensional integrable Fokas–Lenells equation. Proceedings of the XIV International Scientific Conference of Students and Young Scientists "Science and Education – 2019". – Astana, 2019. – P. 260–262.

15. Aizhan A., **Zhassybayeva M.B.,** Yesmakhanova K.R. Lax representation of the dispersionless (2+1)-dimensional Fokas-Lenells equation. Proceedings of the International Conference "Current Problems of Analysis, Differential Equations and Algebra" (EMJ-2019). – Astana, 2019.

16. **Zhassybayeva M.B.,** Abeuova M. Dispersionless limit of the integrable M-LXXII equation. Proceedings of the XX International Scientific Conference of Students and Young Scientists "Science and Education – 2025". – Astana, 2025. – P. 224–226.

17. **Zhassybayeva M.B**. On an integrable Fokas–Lenells equation with a self-consistent source. Proceedings of the XIV International Scientific Conference of Students, Master's Students and Young Scientists "Lomonosov – 2018". – Astana, 2018. – P. 19–21.

18. **Zhassybayeva M.B.,** Myrzakulova Zh.R., Abeuova M. Darboux transformation for the two-layer M-LXXII equation. Academic Journal of Physical and Chemical Sciences. – Almaty, 2025. – No. 4. – P. 24–37.

Connection of the work with research programs:

The work was carried out in accordance with the plans of research activities within the framework of grant funding under the following projects: "Investigation of some problems of astrophysics and cosmology within Einstein and non-Einstein theories of gravity" (2018–2020, No. 0118RK00935, contract No. 03-02-12/1942 dated April 24, 2018, supervisor – R. Myrzakulov); "Investigation of the dynamics of solitons of Fokas–Lenells-type equations" (2025–2027, grant No. AP25794905, supervisor – M.B. Zhassybayeva).

Structure and scope of the dissertation

The structure of the dissertation is determined by the objectives and consists of an introduction, three chapters, and a conclusion. The dissertation contains 108 pages of typed text, 26 figures, and a list of references comprising 137 sources.